# Homework 2

(Due date: February 5th @ 5:30 pm)
Presentation and clarity are very important! Show your procedure!

## PROBLEM 1 (20 PTS)

- In these problems, you MUST show your conversion procedure.
  a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (6 pts)
     ✓ -393.3125, 37.65625, -128.5078125, -31.25.

  b) Complete the following table. The decimal numbers are unsigned: (8 pts.)

| Decimal | BCD | Binary | Reflective Gray Code |
|---|---|---|---|
| 397 | | | |
| | | 1001111010 | |
| | | | 1011100010 |
| 256 | | | |
| | | | 10011100 |
| | | 1110010 | |
| | | | 101011001 |
| | 001010010101 | | |

  c) Complete the following table. Use the fewest number of bits in each case: (6 pts.)

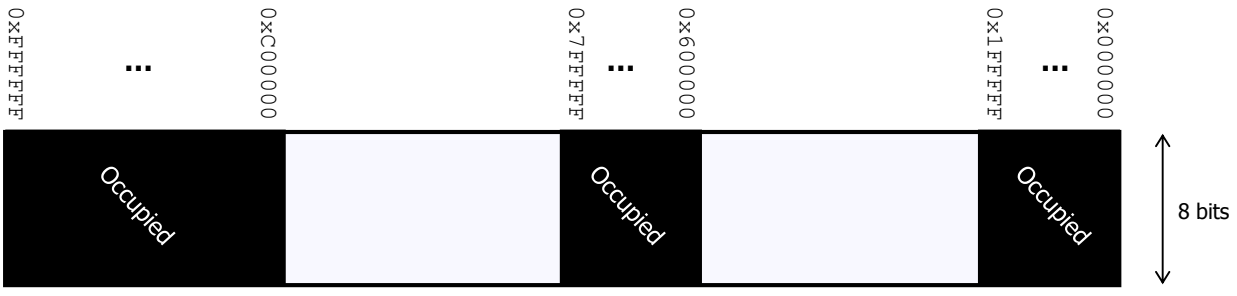
| REPRESENTATION | | | |
|---|---|---|---|
| **Decimal** | **Sign-and-magnitude** | **1's complement** | **2's complement** |
| -123 | | | |
| | | | 100000000 |
| | | 10110010 | |
| | | | 1001101 |
| | 110100101 | | |
| | | 011011001 | |

## PROBLEM 2 (15 PTS)

a) What is the minimum number of bits required to represent: (2 pts)
   ✓ 100,000 symbols?
   ✓ Numbers between 35,000 and 43,192?

b) A microprocessor has a 32-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (5 pts)
   - What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = $2^{10}$ bytes, 1MB = $2^{20}$ bytes, 1GB = $2^{30}$ bytes
   - A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses 0xB1C00000 to 0xB1FFFFFF to this memory device. What is the size (in bytes, KB, or MB) of this memory device? What is the minimum number of bits required to represent the addresses only for this memory device?

c) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (8 pts)
   - What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
   - If we have a memory chip of 2MB, how many bits do we require to address 2MB of memory?
   - We want to connect the 2MB memory chip to the microprocessor. Provide an address range so that 2MB of memory is properly addressed. You can only use the non-occupied portions of the memory space as shown in the figure below.



# PROBLEM 3 (30 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits $n$ to represent both operators. Indicate every carry (or borrow) from $c_0$ to $c_n$ (or $b_0$ to $b_n$). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher byte. (8 pts)

*Example (n=8):*

✓ 54 + 210

| | $c_8$=1 | $c_7$=1 | $c_6$=1 | $c_5$=1 | $c_4$=0 | $c_3$=1 | $c_2$=1 | $c_1$=0 | $c_0$=0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 54 = 0x36 = | | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | + |
| 210 = 0xD2 = | | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | |
| Overflow! → | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | |

✓ 77 - 194

| | Borrow out! → $b_8$=1 | $b_7$=0 | $b_6$=0 | $b_5$=0 | $b_4$=0 | $b_3$=0 | $b_2$=1 | $b_1$=0 | $b_0$=0 | |
|---|---|---|---|---|---|---|---|---|---|---|
| 77 = 0x4D = | | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | - |
| 194 = 0xC2 = | | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | |
| | | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | |

✓ 271 + 137
✓ 111 + 75
✓ 43 - 97
✓ 128 - 43

b) We need to perform the following operations, where numbers are represented in 2's complement: (16 pts)

✓ -97 + 256
✓ 413 + 617
✓ 99 - 62
✓ -127 - 37

- For each case:
  - ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
  - ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
  - ✓ Determine whether there is overflow by:
    i. Using $c_n, c_{n-1}$ (carries).
    ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for $n$ bits, where $n$ is the minimum number of bits for the summands.
  - ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

c) Get the multiplication results of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)

✓ 0100×0101, 1000×0110, 1001×1001.

# PROBLEM 4 (15 PTS)

- In these problems, you can use full adders and logic gates. Make sure your circuit works for all cases. If there is overflow, design your circuit so that the final answer is always the correct one with the correct number of bits.
  a) Given two 4-bit numbers provided in gray code, sketch the circuit that computes the summation of the unsigned decimal numbers these gray codes represent.
  b) Given two 4-bit signed (2's complement) numbers A, B, sketch the circuit that computes $(A - B) \times 4$.

# PROBLEM 5 (20 PTS)

a) Implement the following functions using i) decoders (and OR gates) and ii) multiplexers: (5 pts)

✓ $F = \overline{Y + Z} + XY$
✓ $F = \bar{X} \oplus Y \oplus \bar{Z}$

b) Using only a 4-to-1 MUX, implement the following functions. (5 pts)

✓ $F(X,Y,Z) = \sum(m_0, m_2, m_4, m_6)$
✓ $F(X,Y,Z) = \prod(M_2, M_4, M_5, M_6)$

c) Complete the timing diagram of the circuit shown below: (10 pts)